Differentiation

Question Paper

Course	Edexcel IGCSE Maths
Section	3. Sequences, Functions & Graphs
Topic	Differentiation
Difficulty	Very Hard

Time allowed: 80

Score: /64

Percentage: /100

Question 1a

A curve, C, has equation $y = 2x^2 + 8k^2x - 3$ where k is a constant.

Show that when k = 0, the turning point on C has coordinates (0, -3).

[2 marks]

Question 1b

Show that when $\boldsymbol{k} \neq \mathbf{0}$, the turning point on C must have a negative $\boldsymbol{x}$-coordinate.

[4 marks]

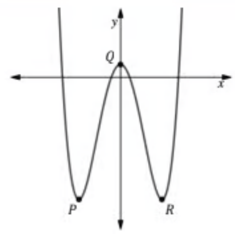
Question 1c

When $k \neq 0$ determine whether or not the y-coordinate of the turning point is negative.

[2 marks]

Question 2

Part of the graph with equation $y = 2x^4 - 16x^2 + 3$ is shown below.

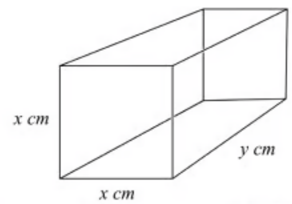


The graph has three stationary points, indicated on the graph by points P, Q and R.
Find the area of the triangle PQR.

[7 marks]

Question 3a

The diagram shows a cuboid with a square cross-section.



The sides of the square face are x cm and the length of the cuboid is y cm.
The cuboid is to have a fixed surface area, A, of 25 cm^2.
Show that the volume of the cuboid, V cm^3 is given by

$$V = \frac{25}{4}x - \frac{1}{2}x^3$$

[4 marks]

Question 3b

Show that the value of x that maximises the volume of the cuboid is $\frac{5\sqrt{6}}{6}$

[4 marks]

Question 3c

Find the maximum volume of the cuboid, correct to 3 significant figures.

[2 marks]

Question 4

A particle P moves along a straight line that passes through the fixed point O
The displacement, x metres, of P from O at time t seconds, where $t \geqslant 0$, is given by

$$x = 4t^3 - 27t + 8$$

The direction of motion of P reverses when P is at the point A on the line.
The acceleration of P at the instant when P is at A is $a \text{ m/s}^2$.
Find the value of a.

a = ...

[5 marks]

Question 5

Two particles, P and Q, move along a straight line.
The fixed point O lies on this line.

The displacement of P from O at time t seconds is s metres, where

$$s = t^3 - 4t^2 + 5t \qquad for\ t > 1$$

The displacement of Q from O at time t seconds is x metres, where

$$x = t^2 - 4t + 4 \qquad for\ t > 1$$

Find the range of values of t where $t > 1$ for which both particles are moving in the same direction along the straight line.

[6 marks]

Question 6

The point A is the only stationary point on the curve with equation $y = kx^2 + \dfrac{16}{x}$ where k is a constant.

Given that the coordinates of A are $\left(\dfrac{2}{3}, a\right)$

find the value of a.
Show your working clearly.

$a =$ ....

[5 marks]

Question 7

The curve $\mathbf{C}$ has equation $y = ax^3 + bx^2 - 12x + 6$ where a and b are constants.

The point A with coordinates (2, −6) lies on $\mathbf{C}$.
The gradient of the curve at A is 16.

Find the y coordinate of the point on the curve whose x coordinate is 3.
Show clear algebraic working.

[6 marks]

Question 8

A particle P is moving along a straight line.
The fixed point O lies on the line.

At time t seconds $(t \geqslant 0)$, the displacement of P from O is s metres where

$$s = t^3 - 9t^2 + 33t - 6$$

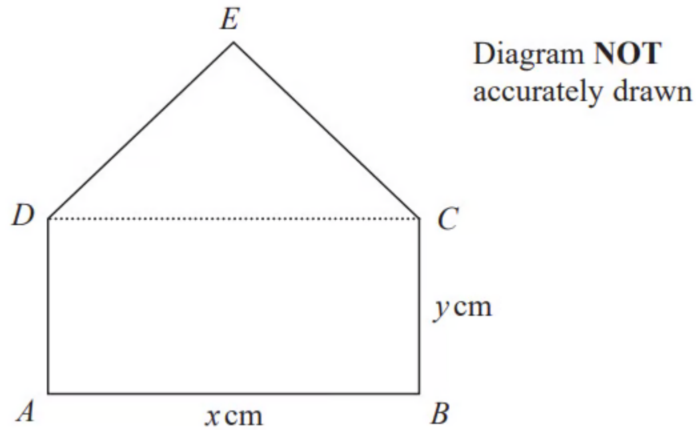
Find the minimum speed of P.

.... m/s

[5 marks]

Question 9a

$ABCED$ is a five-sided shape.



$ABCD$ is a rectangle.
CED is an equilateral triangle.

$AB = x$ cm $\quad BC = y$ cm

The perimeter of $ABCED$ is 100 cm.
The area of $ABCED$ is R cm^2

Show that $R = \frac{x}{4}\left(200 - \left[6 - \sqrt{3}\right]x\right)$

[3 marks]

Question 9b

(i)

Find the value of x for which R has its maximum value.

Give your answer in the form $\dfrac{p}{q-\sqrt{3}}$ where p and q are integers.

$x =$ .... [2]

(ii)

Explain why the maximum value of R is given by this value of x.

[1]

[3 marks]

Question 10

A particle moves along a straight line.
The fixed point O lies on this line.
The displacement of the particle from O at time t seconds , $t \geqslant 0$, is s metres where

$$s = t^3 + 4t^2 - 5t + 7$$

At time T seconds the velocity of P is V m/s where $V \geqslant -5$

Find an expression for T in terms of V.

Give your expression in the form $\dfrac{-4 + \sqrt{k + mV}}{3}$ where k m and are integers to be found.

$T =$ ....

[6 marks]